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Abstract: This paper is focused on the influence of geometry on the combination of inter-cell and intra-cell interferences in the downlink of large cdma networks. We use an exact representation of the geometry of the downlink channels to define scalable admission and congestion control schemes, namely schemes that allow each base station to decide independently of the others what set of voice users to serve and/or what bit rates to offer to elastic traffic users competing for bandwidth. We then study the load of these schemes when the size of the network tends to infinity using stochastic geometry tools. By load, we mean here the distribution of the number of voice users that each base station can serve and that of the bit rate offered to each elastic traffic user.

Key-words: CDMA, downlink, admission control, congestion control, Poisson-Voronoi model

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Contrôle d'admission/congestion et charge maximale sur la voie descendante de grands réseaux CDMA

Résumé : Cet article porte sur la géométrie des interférences intra-cellulaires et extra cellulaires de la voie descendante des grands réseaux CDMA. Nous utilisons une représentation exacte de cette géométrie pour définir des algorithmes extensibles de contrôle d'admission et de contrôle de congestion. L'extensibilité de ces algorithmes vient du fait que chaque station de base peut décider indépendamment des autres quel ensemble d'utilisateurs de type voix elle est en mesure d'accepter et aussi quels débits offir aux utilisateurs apportant un trafic élastique. La charge maximale autorisée par ces algorithmes de contrôle est ensuite évaluée dans le cas d'un réseau infini au moyen d'outils de géométrie aléatoire. Par charge, on entend ici la distribution du nombre des utilisateurs de type voix que chaque station de base peut accepter et celle du débit qu'elle peut offir à chaque utilisateurs apportant un trafic élastique.

Mots-clés : CDMA, voie descendante, contrôle d'admission, contrôle de congestion, géométrie aléatoire, mosaïque de Poisson-Voronoi

1 Introduction

This paper concerns the evaluation of the load of the downlink (forward channel) of CDMA networks, with a special emphasis on the limitations of load due to inter-cell and own-cell interferences. More precisely we analyze the maximal number of customers that such a network can serve at a given bit rate and/or the maximal bit rates that such a network can provide to a given customer population.

The quantification of these load constraints is shown to allow one to define

- *Admission control policies* in the case of predefined customer bit rates (e.g. voice); i.e., schemes allowing one to decide whether a new customer can be admitted or should be rejected as its admission could make the *global* power allocation problem unfeasible;

- *Congestion control policies* in the case of customers with elastic bit rates (e.g. data); i.e., schemes allowing one to determine the maximal fair customer bit rates that preserve the feasibility of the power control problem at any time, in function of the customer population in all cells at this time.

The main practical aims of the present paper are to propose decentralized and scalable admission and congestion control protocols that guarantee that the network remains in a position to solve the power allocation problem at any time and to evaluate the two notions of load alluded to above, via mathematical analysis and simulation.

The evaluation part relies on a model which uses planar point processes and stochastic geometry. Indeed, the model has several key components, the spatial location pattern of base stations (BS's), the spatial location pattern of users, the attenuation (path loss) function and the policy of assignment of users to BS's, which are geometry-dependent, in addition to the non-geometric components such as orthogonality factors, pilot signals and external noise.

We will allow both patterns of locations to be countably infinite so as to address the *scalability* questions, and to check the ability of the proposed algorithms to continue to function well as the size of the network goes to infinity.

The basic assignment policy will be that where each mobile is served by the closest BS. It is basically equivalent to the optimal-SIR-choice scheme and to the honeycomb model in the classical hexagonal case.

This model will be studied under various stochastic settings, which take into account the irregularities of both the infrastructure and the traffic, in a statistical way. Mimicking Kendall's notation in queueing theory, we can say that our most general stochastic model is G/G, where G means general ergodic spatial point process; the first G stands for the process of BS's and the second for the process of mobiles (BS/Mobiles). As special cases we consider M/M and D/M and in particular H/M, where M means, possibly inhomogeneous Poisson, D general deterministic periodic, H hexagonal.

The new analytical results on these notions of load are of independent interest as these notions can be viewed as surrogates of information theoretic definitions of capacity. (Note that we use the notion "load" for the number of users served and keep the notion "capacity" reserved for information-theoretic description that concerns bit-rates pumped by the network.) Their mathematical tractability (at least in some particular cases) opens new ways of assessing e.g. CDMA networks optimal design and economic planning.

The paper is organized as follows. We first give a brief survey of the literature in § 2. § 3 revisits the algebra of power allocation. The admission and congestion control algorithms are introduced in § 4 and § 5 respectively; some implementation issues are discussed in § 6. The stochastic geometry models allowing one to evaluate load are introduced in §7 and studied in § 8. Simulation and numerical results are gathered in § 9. Proofs of the statements of § 3 can be found in Appendix.

2 Situation

The problem of CDMA capacity (load) constraints has already been considered by several authors. Nettleton and Alavi [16] first considered the power allocation problem in the cellular spread spectrum context.

In Gilhousen et al [6], the problem was posed in the following way. Suppose Base Station number 1 (BS 1) emits at the total power S_1 in the presence of $K-1$ other BS's, which emit at power $S_2, \ldots, S_K$ respectively. How many users N_1 can then BS 1 accommodate assuming that the load of the network is only interference-limited and that each user has a required bit rate of W? The sufficient condition (and thus conservative load constraint) proposed in [6] reads

$$\sum_{i=1}^{N_1} C_i\left(1+\sum_{k=2}^{K}\frac{(S_k)_i}{(S_1)_i}+\frac{\eta}{(S_1)_i}\right)\le 1\,. \tag{2.1}$$

In this formula, $(S_k)_i$ the power received by user i from BS k and $C_i = (E_b/N_0)_i/(\beta W/R)$, where $(E_b/N_0)_i$ is the bit energy-to-noise density ratio of user i, and R, β, η are the bandwidth, the fraction of the total power devoted to the pilot signal and the external noise, respectively.

This simple condition allows for the determination of N_s but it does not reflect a key feature, which is the competition of BS's for allocating powers to their users: in reality the total power emitted by the BS should depend on the number of users, namely S_k should be a function $S_k(N_1, \ldots, N_K)$.

In order to address this issue, Zander [20, 19] expresses the global power allocation problem by the multidimensional linear inequality

$$\mathbb{Z}\mathbf{S} \le \frac{1+C}{C}\mathbf{S} \tag{2.2}$$

with unknown vector $\mathbf{S}$ of emitted powers; here one assumes the required signal-to-interference ratio C (or equivalently the required user bit rate) to be given and one assumes the matrix $\mathbb{Z}$, the i,k-th entry of which gives the normalized path losses/gain between user i and BS k, to be given too. The main result is then that the power allocation is *feasible* if there exists a non-negative, finite solution to (2.2); the necessary and sufficient condition is that $C \le 1/(\lambda^*-1)$, where λ^* is the Perron-Frobenius eigenvalue of the (positive) matrix $\mathbb{Z}$. In order to simplify the problem, all same-cell channels are assumed to be completely orthogonal and the external noise is suppressed. In [20] the issue of decentralization for narrow-band systems is also addressed.

Foschini and Miljanic [5] and Hanly [8] introduced external noise to the model: Foschini considered a narrow-band cellular network and Hanly a two-cell spread spectrum network. On the basis of the previous works, in several articles Hanly extended the model. Hanly [10] extends this approach to the case with in-cell interference and external noise (essentially for the uplink). Using the block structure of $\mathbb{Z}$, he solves the problem in two steps: first the own-cell power allocation conditions are studied (microscopic view) and then the macroscopic view considers some aggregated cell-powers. He also interprets λ^* as a measure of the traffic congestion in the network.

The evaluation of λ^* can be done either from a centralized knowledge of the state of the network, which is non practical in large networks, or by channel probing as suggested in § VIII of [10] and described in [21]. When it exists, the minimal finite solution of inequality (2.2) can also be evaluated in a decentralized way (using Picard's iteration of operator $\mathbb{Z}$, cf. the discussion in § IX of [9]). However this does not provide decentralized admission or congestion control algorithms, namely scalable ways of controlling the network population or bit rates in such a way that the power allocation problem remains feasible, namely that λ^* remains less than 1.

In the present paper, we continue the approach of [20, 10] and propose decentralized admission congestion control protocols for this context, based on a simple mathematical fact saying that the maximal eigenvalue of any sub-stochastic matrix (matrix with nonnegative entries, whose row sums are less than 1) is less than 1.

We find that this, when applied to the downlink power allocation problem, takes a form similar to (2.1), with the powers received $(S_k)_i$ replaced by path losses/gains from BS k to user i. Since path loss basically depends on the geometry only and neither on the number of users served nor on the powers emitted, our version of Equation (2.1) no longer depends implicitly on N_k.

There is also a rich literature on the single BS case. For recent advances on the downlink case see e.g. [14], [15].

Finally, we remark that our downlink load analysis is essentially different from a typical approach to the uplink load, where the main assumption says that all the mobiles are received by their BS's at the same level due to a perfect power adaptation (see e.g. [11]).

3 Power Allocation Algebra Revisited

In this section, we remind the power allocation problem and its reduction to a linear algebraic setting following the lines of [10].

Let $\mathcal{N}_{BS} = \{Y^j\}_j$, $Y^j \in \mathbb{R}^2$ be the locations of the base stations (BS) in the plane (most results below extend to the d-dimensional Euclidean space). Suppose that the BS located at Y^j is to serve a set of mobiles located at $\mathcal{N}_M^j = \{X_i^j\}_i$, $X_i^j \in \mathbb{R}^2$ at some given SINR C_i^j. Denote the power loss of the signal (attenuation, fading, etc.) on the path from y to x, $x, y \in \mathbb{R}^2$ by $l(y, x)$. We assume also some local external noise W_i^j at X_i^j and pilot power P_j emitted by each BS j.

The network can handle this population of mobiles on the downlink if each BS Y^j can allocate some power S_i^j to mobile X_i^j, s.t. the following set of inequalities on SINR is satisfied: for all i, j,

$$\frac{S_i^j l(Y^j, X_i^j)}{W_i^j + (I_i^j)_{own} + (I_i^j)_{ext}} \geq C_i^j. \tag{3.1}$$

In this equation, $(I_i^j)_{own} = \kappa_j l(Y^j, X_i^j)(P_j + \sum_{i' \neq i} S_{i'}^j)$, and $(I_i^j)_{ext} = \gamma \sum_{k \neq j} l(Y^k, X_i^j)(P_k + \sum_{i'} S_{i'}^k)$, where $\kappa_j \geq 0$ is the own-cell orthogonality factor in cell j, whereas $\gamma \geq 0$ is the other-cell orthogonality factor. (Our results will cover the special cases $\kappa_j = 0$ and/or $\gamma = 1$.)

The set of inequalities (3.1) is equivalent to some linear matrix inequality (understood coordinate-wise)

$$\tilde{\mathbf{S}} \geq \tilde{\mathbb{A}}\tilde{\mathbf{S}} + \tilde{\mathbf{b}}, \tag{3.2}$$

where $\tilde{\mathbf{S}} = \left((S_i^j)_{i=1}^{\#\mathcal{N}_M^j} \right)_{j=1}^{\#\mathcal{N}_{BS}}$ is the unknown vector of all individual powers. So one can solve the power allocation problem if and only if one can find a finite solution to this set of linear inequalities. Of course this solution might be non-implementable if the sum of the powers required on the downlink of a given BS exceeds its maximal power. As in [9], we will in a first step ignore the effects of transmitter power constraints. We will return to this question in 6.1 and ignore these extra limitations in what follows.

Before studying the existence of finite solutions of this set of linear inequalities, we use its block structure to obtain some necessary conditions.

Proposition 3.1 *Inequality (3.2) is equivalent to the following two-step problem:*

- *solving some linear inequality*

$$\mathbf{S} \geq \mathbb{A}\mathbf{S} + \mathbf{b} \tag{3.3}$$

for the total powers $\mathbf{S} = (S_j)_{j=1}^{\#\mathcal{N}_{BS}}$, $S_j = \sum_i S_i^j$ *emitted by the BS's on traffic channels,*

- *and for a given solution* $\mathbf{S} = \left(S_j\right)_j$ *of (3.3), for each j, solving some linear inequality*

$$\tilde{\mathbf{S}}_j \geq \tilde{\mathbb{A}}_{jj}\tilde{\mathbf{S}}_j + \tilde{\mathbf{d}}_j \tag{3.4}$$

for the individual powers $\tilde{\mathbf{S}}_j = (S_i^j)_{i=1}^{\#\mathcal{N}_M^j}$ *on the traffic channels of BS j, under the constraint* $\sum_i S_i^j = S_j$.

We will call (3.3) the *global power allocation problem* and (3.4) the *local* or *in-cell power allocation problem.* The precise definition of the last vectors and matrices, as well as conditions under which there exist finite solutions to these problems are given below. The proofs can be found in Appendix.

3.1 Local Power Allocation Problem

Let

$$H_i^j = \frac{C_i^j}{1 + \kappa_j C_i^j}. \tag{3.5}$$

The local power allocation problem (3.4) for the jth BS reads: for all i

$$\begin{aligned} S_i^j \;\; \geq \;\; & H_i^j \Bigg(\frac{W_i^j}{l(Y^j, X_i^j)} + \kappa_j (S_j + P_j) \\ & + \gamma \sum_{k \neq j} \frac{l(Y^k, X_i^j)}{l(Y^j, X_i^j)} (P_k + S_k) \Bigg). \end{aligned} \tag{3.6}$$

Proposition 3.2 *Suppose a collection of non-negative, finite aggregated powers $\{S_j\}$ satisfying* (3.3) *is given. The following inequality*

$$\kappa_j \sum_i H_i^j < 1 \tag{3.7}$$

is necessary and sufficient for the existence of a positive and finite solution of (3.6) *under the constraint* $\sum_i S_i^j = S_j$. *If it holds then for all* i

$$S_i^j = H_i^j S_j + d_i^j + \xi_i \,, \tag{3.8}$$

where

$$d_i^j = H_i^j \left(\frac{W_i^j}{l(Y^j, X_i^j)} + \kappa_j P_j \sum_{k \neq j} \frac{l(Y^k, X_i^j)}{l(Y^j, X_i^j)} (P_k + S_k) \right)$$

and any collection of non-negative $\{\xi_i\}_i$ *satisfying* $\sum_i \xi_i = S_j - \left(\mathbb{A}\mathbf{S}\right)_j - b_j$, *with the elements of* $\mathbb{A}$ *given by (3.9)–(3.10), and* b_j *the elements of* $\mathbf{b}$ *given by (3.11).*

3.2 Global Power Allocation Problem

In view of Proposition 3.2, we assume from now on that (3.7) holds for all j; i.e., that the local power allocation problem can be solved, and we concentrate on the global problem (3.3), with $\mathbb{A} = (a_{jk})$, $\mathbf{b} = (b_j)$ given by

$$a_{jj} = \sum_i \kappa_j H_i^j \,, \tag{3.9}$$

$$a_{jk} = \gamma \sum_i \frac{H_i^j l(Y^k, X_i^j)}{l(Y^j, X_i^j)}, \quad k \neq j \tag{3.10}$$

$$b_j = \sum_i H_i^j \left(\frac{W_i^j}{l(Y^j, X_i^j)} + \kappa_j P_j + \gamma \sum_{k \neq j} \frac{l(Y^k, X_i^j)}{l(Y^j, X_i^j)} P_k \right). \tag{3.11}$$

As it was mentioned for (3.2), the existence of solutions of (3.3) depends on the spectral radius of the possibly infinite matrix $\mathbb{A}$. We remind briefly some notions of the theory on the matter in Appendix.

Let us denote by $\mathbb{A}^n = (a_{jk}^n)_{jk}$ the nth power of $\mathbb{A}$, with $\mathbb{A}^0 = \mathbb{I}$ being the identity matrix. Moreover, let $\mathbb{A}^* = (a_{jk}^*)_{jk} = \sum_{n=0}^{\infty} \mathbb{A}^n$. Note that $\mathbb{A}^n$ for each $n \geq 0$ and $\mathbb{A}^*$ are well defined, but they may have some or all their entries infinite. Assume now $\gamma > 0$ and $C_i^j > 0$. If $\#\mathcal{N}_M^j > 0$ then $a_{jk}^n > 0$ for all $n > 1$. Thus excluding BS's serving no mobiles, we get a positive (and therefore irreducible) matrix $\mathbb{A}$ for which all the power series $A_{jk}(z) = \sum_{n=0}^{n} a_{jk}^n z^n$ for $j, k = 1, 2, \ldots$ have a common convergence radius $0 \leq R < \infty$ called the *convergence radius* of $\mathbb{A}$. Moreover $A_{jk}(R) < \infty$ for all $j \neq k$, and $A_{jj}(R)$ is finite or infinite at the same time for all j making $\mathbb{A}$ respectively *transient* or *recurrent.*

Proposition 3.3 *A necessary condition for the global power allocation problem* (3.3) *to have a positive finite solution is that the convergence radius* R *of* $\mathbb{A}$ *be greater than or equal to 1. In case of equality,* $\mathbb{A}$ *has to be transient. Then any solution of (3.3) is of the form* $\mathbb{A}^*(\mathbf{b} + \boldsymbol{\xi}) + \mathbf{z}$, *where* $\boldsymbol{\xi} \geq 0$ *and* $\mathbf{z} \geq 0$ *s.t.* $\mathbf{z} = \mathbb{A}\mathbf{z}$, *with the last term existing only in infinite-dimensional case.*

It may happen that $\mathbb{A}^*$ has all its entries finite and that the minimal solution $\mathbb{A}^*\,\mathbf{b}$ has all its entries infinite. Note that any solution $\mathbf{S} = (S_j)$ of (3.3) has the following *coordinate-wise solidarity property*: if for any j, $S_j = \infty$, then $S_j = \infty$ for all j.

Note that the successive iterations Ψ^n of the linear operator Ψ on $\mathbb{R}^{\#\mathcal{N}_{BS}}$, defined by $\Psi(\mathbf{s}) = \mathbb{A}\mathbf{s} + \mathbf{b}$ tend coordinate-wise with $n \to \infty$ to a solution $\mathbb{A}^*\mathbf{b} + \mathbf{z}$ and $\Psi^n(\mathbf{0})$ tend (increase) to the minimal solution.

The following result gives two simple conditions: a sufficient condition and a necessary one for the convergence radius R of $\mathbb{A}$ to be greater that 1.

Proposition 3.4 *If matrix $\mathbb{A}$ is strictly* substochastic *(i.e. for each j, $\sum_k a_{jk} \le 1$ and for some j_0, $\sum_k a_{jk} < 1$), then $R < 1$. On the other hand if $\mathbb{A}$ has a* super-stochastic *block (i.e. $\sum_{k\ in\ block} a_{jk} \ge 1$ for all j in a block and $\sum_{k\ in\ block} a_{jk} > 1$ for some j), then $R < 1$.*

4 Decentralized Admission Control Protocol

Assume the bit rates of all users (or equivalently all C_i^j parameters) to be specified. The admission control problem can then be posed as follows: for a given mobile population $\mathcal{N}_M^j$, for all j, check whether $R > 1$. Indeed, if this holds, all mobiles can be served on the downlink, whereas if $R = 1$ and $\mathbb{A}$ is recurrent then (3.3) has a finite solution only if the external noise $W_i^j = 0$ and $P_j = 0$ for all i, j, and if $R < 1$, then the global allocation problem has no positive finite solution. If $R \le 1$, one should then decrease the population $\mathcal{N}_M^j$, that is, one should admit only some subset $\widetilde{\mathcal{N}}_M^j \subset \mathcal{N}_M^j$, for some or all j, s.t. the corresponding convergence radius $\widetilde{R}$ is strictly larger than 1. The main difficulties with this scheme are twofold:

- First the computation of the convergence radius requires the knowledge of the whole $\mathbb{A}$ matrix, that is some centralized knowledge not only of fixed (or slowly varying) data such as the positions of all BS's, but in addition that of quickly varying variables such as the the positions of all mobiles in all cells. In the *finite dimension* case, the fact that $R = \lim_{n\to\infty} a_{jk}^n / a_{jk}^{n+1}$ for all j, k, implies

$$\frac{(\Psi^n(\mathbf{s}))_j - (\Psi^{n-1}(\mathbf{s}))_j}{(\Psi^{n+1}(\mathbf{s}))_j - (\Psi^n(\mathbf{s}))_j} \to R \quad \text{as} \quad n \to \infty, \tag{4.1}$$

  which leads to some potential "on line" estimation of R by channel probing as mentioned in § VIII of [10] and described in [21]. Notice that (4.1) may not hold in the infinite dimensional case, which indicates that the quality of this estimation may not be satisfactory for large networks.

- If the population is s.t. the power allocation problem is not feasible, this estimation of R does not lead to procedures allowing one to determine population or bit rate reductions that might lead to a feasible allocation problem.

4.1 DACP Principles

The admission control protocol described in this section is based on the sub-stochastic condition on the matrix $\mathbb{A}$ mentioned in Proposition 3.4. Let

$$f_i^j = \kappa_j H_i^j + \gamma \sum_{k \neq j} \frac{H_i^j l(Y^k, X_i^j)}{l(Y^j, X_i^j)}, \; i \in \mathcal{N}_M^j. \tag{4.2}$$

Note that matrix $\mathbb{A}$ is sub-stochastic if and only if

$$\sum_{i \in \mathcal{N}_M^j} f_i^j \leq 1, \tag{4.3}$$

for all j, with strict inequality for some j. The last condition is only sufficient and non-necessary for the existence of solutions of the problem and thus in principle it only gives a lower bound on the load of the system.

Note that (4.3) has a form similar to (2.1), with the powers received $(S_k)_i$ replaced by path losses/gains on the distance form BS k to user i. Since path losses basically depend on the geometry only and neither on number of users served nor on the powers emitted, our equivalent of Equation (2.1) no longer depends on N^k.

Also note that Condition (4.3) depends not only on the number of mobiles in the cell of BS j, but also on their locations in the cell. In case these users do move within the cell, this mobility alone may lead to customer rejection.

Our interest in Condition (4.3) comes from the fact that it leads to the following algorithm:

Decentralized Admission Control Protocol (DACP): *Each BS checks periodically whether Condition* (4.3) *is satisfied and if not, enforces it by reducing the population* $\mathcal{N}_M^j$ *of its mobiles to some subset* $\widetilde{\mathcal{N}}_M^j$ *s.t. inequality* (4.3) *holds when* $\mathcal{N}_M^j$ *is replaced by* $\widetilde{\mathcal{N}}_M^j$. *When a new mobile user applies to some BS, the BS accepts it if Condition* (4.3) *is satisfied with this additional user and rejects it otherwise.*

The decentralized nature of these control schemes stems from the fact that for each j, inequality (4.3) depends on the characteristics of the mobiles in cell j but not on the location or number of the mobiles of $\mathcal{N}_M^k$ for $k \neq j$. So station j can perform this check or this reduction independently of the others, based on the sole knowledge of slowly varying global data such as the locations of the other BS's.

4.2 DACP Reduction Schemes

If a given pattern of users yields a sum (4.3) larger than 1, then a reduction of mobiles should be applied.

Here are some natural reduction candidate algorithms which reflect the *load versus coverage tradeof.*

1. *An admission algorithm that maximizes the number of mobiles served by each BS*: station j ranks the mobiles of $\mathcal{N}_M^j$ according to the cost function f_i^j and serves the largest subset of $\mathcal{N}_M^j$ obtained when starting with the mobiles having the smallest cost. This clearly maximizes the total number of mobiles served by the network under this admission control scheme. The drawbacks of this are clear. For instance in the case when all thresholds C_i^j are the same, small cost mobiles are those in the vicinity of the BS; in other words, the coverage of this scheme is poor as mobiles that are at the edge of the cell are discriminated against. In addition, since the positions of mobiles change over time, this scheme might lead to the interruption of some communications e.g. due to the arrival of a newcomer located in the vicinity of the BS.

2. *An admission algorithm that uniformizes coverage*: Station j admits customers of $\mathcal{N}_M^j$ in a random order until inequality (4.3) is satisfied. The main advantage of this scheme is

that since customers are not admitted based on their location, it ought to lead to a more uniform coverage, and also to a better robustness to mobility (see §7).

Admission control schemes should try to take into account other goals as e.g. protection of calls in progress.

5 Decentralized Congestion Control Protocol

In this section, we do not assume the bit rates of users (or equivalently the C_i^j parameters) to be specified. We are interested in a scheme for elastic traffic, namely for traffic which can accommodate bit rate variations.

We first consider the case with no admission control, where an increase of the number of users in a cell is just coped with via a reduction of the bit rates of the users of this cell, like in TCP where the increase of the number of competitors eventually results in a decreased bit rate for all, and where no user is ever rejected.

We will look for fair schemes, where all users in a given cell are supposed to have the same bit rate, namely the same SIR $\mathcal{C}^j$ for all users in the cell of BS j. If the mobile population $\mathcal{N}_M^j$ is fixed for all j, then (3.3) reads

$$\frac{\mathcal{C}^j}{1+\kappa_j\mathcal{C}^j}\sum_{i\in\mathcal{N}_M^j}\left(\kappa_j+\gamma\sum_{k\neq j}\frac{l(Y^k,X_i^j)}{l(Y^j,X_i^j)}\right)\leq 1, \tag{5.4}$$

for all j, with strict inequality for some j. That is, the maximal fair SIR that BS j can offer to its users is

$$\mathcal{C}^j=\frac{1}{\left(\kappa_j(\#\mathcal{N}_M^j-1)+\gamma\sum_{i\in\mathcal{N}_M^j}\sum_{k\neq j}\frac{l(Y^k,X_i^j)}{l(Y^j,X_i^j)}\right)_+} \tag{5.5}$$

where x_+ is $\max(x,0)$. Using a Gaussian channel approximation, one can deduce the maximal fair bit rate offered to users of cell j is

$$\mathcal{B}^j=B\log(1+\mathcal{C}^j), \tag{5.6}$$

with B the CDMA channel bandwidth and with $\mathcal{C}^j$ the quantity defined above. Even when the number of mobiles in the cell of BS j does not vary, the denominator of (5.5) varies with time when users move.

5.1 DCCP Principles

Decentralized Congestion Control Protocol (DCCP): Each BS periodically allocates the fair rate given by Equation (5.6) *to all mobiles in its cell. This fair rate is also updated at any time when a customer joins or leaves the cell.*

6 Variants and Implementation Issues

6.1 DACP/DCCP with Security Margin

One can solve the power allocation problem if and only if the convergence radius R of matrix $\mathbb{A}$ is greater than 1. However the solution might be non-implementable if the sum of the powers that a BS has to allocate to its mobiles exceed the maximal power of the BS. It is not the purpose of the present paper to investigate the effects of transmitter power constraints in detail. However we will make the following simple observation. Suppose that the model is perfectly periodic, i.e., all the lines of $\mathbb{A}$ are identical $a_{jk} = a_{1k}$. This makes $a^n_{jk} = a_{1k}(\sum_i a_{1i})^n$ and $a^*_{jk} = a_{1k}\sum_{n=0}^{\infty}(1/R)^n$, where the convergence radius $R = 1/\sum_i a_{1i}$ (note by the way, that in this case substochasticity is equivalent to $R > 1$). Consequently, for all j

$$S_j = S_1 = \sum_k a^*_{1k} b_k = \frac{1}{1-1/R}\sum_k a_{1k} b_k \,.$$

Thus the global power allocation solution $\mathbf{S}$ is sensitive to $\mathbf{b}$ if R is close to 1, which may lead to situations where the minimal solution exceeds the maximal power of a BS.

Due to this fact, it may by useful to introduce some *security margin* in the congestion control, namely imposing a stronger condition $R > 1+\delta$ for some $\delta > 0$. In this case, the DACP/DCCP condition takes the form $\sum_{i\in\mathcal{N}^j_M} f^j_i \le 1-\delta$.

6.2 DACP/DSCP Implementation

For each mobile $i \in \mathcal{N}^j_M$ the two terms it contributes to the sum (4.3) can be rewritten as follows: $f^j_i = H^j_i$(total path loss of user i)/(own-BS path loss of user i). Although the SIR threshold of a mobile H^j_i is known to the BS, the ratio of the path losses is not. This could be measured by the user as the (inverse) of the SIR ratio on a "virtual" isolated common channel on which *all the BS transmit with the same power.* The pilot channels could possibly be used to estimate these path losses. The path-loss measurements can then be transmitted to the BS when the mobile applies for access, and periodically updated. From the knowledge of f^j_i for all i in its cell, and that of a newcomer, the BS can judge, before trying to allocate powers, whether it is possible take the newcomer, and this regardless the populations of users served by other BS (provided they all apply DACP).

6.3 DACP and "Join the Less Loaded BS"

A natural modification of the DACP is that where each mobile joins the BS that is less loaded. Suppose that each BS broadcasts the current value Σ^j of its sum (4.3). Then each newcomer able to calculate its f^j_i factor with respect to neighboring BS's is in a position to choose the BS minimizing the sum $\Sigma^j + f^j_i$, provided this is less than 1.

There are of course interesting implementation issues pertaining to the broadcasting of the Σ^j sums that we will not address here.

6.4 Mixing DACP and DCCP

One can adapt the last algorithms to the case of a mix of fixed and elastic bit rate populations, which will be denoted by $\mathcal{N}^j_F$ and $\mathcal{N}^j_E$ respectively for cell j.

A first algorithm that requires little control overhead is that where a proportion $0 < \beta^j < 1$ of the budget of BS j is reserved for fixed bit rate, which leads to the admission of a subset $\widetilde{\mathcal{N}}^j_F$ of $\mathcal{N}^j_F$ s.t. $\sum_{i\in\widetilde{\mathcal{N}}^j_F} f^j_i \le \beta^j$, whereas the complement $1-\beta^j$ is reserved for elastic traffic, which leads to a fair share SIR of

$$C^j = \frac{1-\beta^j}{\left(\kappa_j(\#\mathcal{N}^j_E - 1) + \gamma\sum_{i\in\mathcal{N}^j_E}\sum_{k\neq j}\frac{l(Y^k,X^j_i)}{l(Y^j,X^j_i)}\right)_+} \tag{6.7}$$

for each elastic traffic mobile.

A second and more adaptive algorithm is that where the elastic traffic mobiles use the part of fixed rate budget that is not used (if any); in this case, the fair share SIR of each elastic traffic mobile is

$$C^j = \frac{1 - \sum_{i \in \widetilde{\mathcal{N}}_F^j} f_i^j}{\left(\kappa_j(\#\mathcal{N}_E^j - 1) + \gamma \sum_{i \in \mathcal{N}_E^j} \sum_{k \neq j} \frac{l(Y^k, X_i^j)}{l(Y^j, X_i^j)}\right)_+}. \tag{6.8}$$

The last version of the algorithm require that BS j broadcast the value of $\sum_{i \in \widetilde{\mathcal{N}}_F^j} f_i^j$ on some periodic basis.

One can of course consider versions of these algorithms with security margin.

7 Mathematical Analysis of the Model

Suppose the pattern of BS's is fixed and assume that each BS applies DACP. Assume also that some parametric stochastic model is given for the population of mobiles, with some spatial intensity say λ_j in cell j. Then the probability with which (4.3) holds for BS j (i.e., the probability with which BS j can accept all the mobiles in its cell under DACP) is a BS-centered measure of the admission level in this cell. User-centered admission level measures, like the frequency of call admissions in cell j, can then also be derived.

One can also invert the problem and define the admission load of cell j as follows: given a BS-centered or user-centered QoS level, the admission load of cell j within this parametric context can then be defined as the maximal intensity λ_j s.t. (4.3) is satisfied with a large enough probability.

Similar questions can be formulated on DCCP as well. Here a stochastic setting will allow one to define the law of the fair rate obtained by the mobiles of a given cell. The inverse problem is that where one asks for the maximal mobile intensity that allows one to guarantee a fair bit rate higher than X with a large enough probability.

Making the above notions precise requires some parametric setting for the geometry-dependent components of the model, and in particular of the patterns. The next subsection will discuss some of the possible settings.

7.1 Periodic Models – Classical Calculations Revisited

In order to retrieve from our model the classical load (capacity) calculations, we have to assume the following ideally periodic model. Suppose that the BS's $\{Y^j\}$ are located in space according to a perfect (e.g. hexagonal) grid and suppose that each hexagonal cell has the same number N of mobiles, and that these N mobiles are located exactly in the same way with respect to each BS. Assume that all parameters of our model are constant i.e. $C_i^j \equiv C$, $W_i^j \equiv W$, $\kappa_j \equiv \kappa$, and $P_j = 0$ for simplicity; if one looks for constant solutions $S_j \equiv S$ of the global power allocation problem. Then (3.3) written for the BS (say 0) located at the origin, reduces to

$$S\Big(1 - NH(\kappa + \gamma I_2)\Big) \geq NWI_1H\,, \tag{7.1}$$

where $H = C/(1+\kappa C)$,

$$I_1 = 1N \sum_i \frac{1}{l(0, X_i^0)}\,,$$

and

$$I_2 = \frac{1}{N} \sum_{k \neq 0} \sum_i \frac{l(Y^k, X_i^0)}{l(0, X_i^0)}.$$

It is easy to see that (7.1) has a positive solution if and only if

$$N < \frac{1}{\kappa H (1 + \gamma I_2/\kappa)}. \tag{7.2}$$

Note that $N < H$ is the well know pole (one cell) capacity Condition (3.7) for the ideal model and $\gamma I_2/\kappa$ is a correcting term traditionally called *other-cell-to-in-cell interference ratio.*

The last model is unsatisfactory for several reasons, and in particular because of the regularity of both the antenna and the user patterns. In the next sections we propose stochastic models allowing one to relax this assumption.

7.2 Stochastic Models

In the remaining part of this section we give a review of some possible stochastic settings for the general model and we discuss probabilistic properties of the power allocation problem with and without admission or congestion control. Specifically, we model locations with point processes, and powers, noises, SINR's with random variables so as to capture the space-time variability of configurations. In contrast to the cases typically considered in the literature, we are primarily interested in models on the whole plane that allow one to address scalability. In what follows, we will consider the case when $l(y, x) = L(x - y)$.

7.2.1 General Stationary Ergodic Model (G/G)

Suppose $\tilde{\mathcal{N}}_{BS} = \left\{ \left(Y^j, \mathcal{N}_M^j, \{C_i^j, W_i^j\}_i, \kappa_j \right) \right\}_j$ is a general stationary ergodic marked point process on $\mathbb{R}^2$.

Proposition 7.1 *The power allocation problem* (3.1) *in the model driven by a general ergodic process* $\tilde{\mathcal{N}}_{BS}$ *has a positive and finite solution with probability either 0 or 1.*

Proof: Note that the events { inequality (3.7) holds for all j }, $\{\mathbb{A}^* \mathbf{b} < \infty\}$, { convergence radius $R \leq 1$ }, { convergence radius $R > 1$ }, { matrix $\mathbb{A}$ is transient }, { matrix $\mathbb{A}$ is recurrent } are invariant with respect to the discrete shift of $\tilde{\mathcal{N}}_{BS}$ in $\mathbb{R}^2$. Thus each of them has probability 0 or 1. In view of Propositions 3.1–3.3, the event that (3.1) has a positive and finite solution can be expressed by means of standard boolean operations on the above events. ■

Note that the convergence radius R of the random matrix $\mathbb{A}$ in the general ergodic model is deterministic. We will call the property expressed in Proposition 7.1 *(stochastic) solidarity* of the solution of the power allocation problem.

7.2.2 The Homogeneous Poisson-Voronoi Model (M/M)

Suppose now that $\mathcal{N}_{BS} = \{Y^j\}$ is a homogeneous Poisson point process with intensity $0 < \lambda_{BS} < \infty$. Assume that the sequence $(\{C_i^j, W_i^j\}_i, \kappa_j)$ is made of independent and identically distributed (i.i.d.) random variables, and suppose that for each j the sequence (C_i^j, W_i^j) is also i.i.d. in i and independent of κ_j.

Suppose moreover that the point process $\mathcal{N}_M$ describing the locations of all mobiles is another independent homogeneous Poisson point process with intensity $0 < \lambda_M < \infty$. Let the pattern $\mathcal{N}_M^j$ of mobiles served by the BS located at Y^j be the set of points of $\mathcal{N}_M$ located in the Voronoi cell of point Y^j w.r.t. the point process $\mathcal{N}_{BS}$; i.e., $\mathcal{N}_M^j = \mathcal{N}_M \cap V^j(\mathcal{N}_{BS})$, for all j, where

$$V^j(\mathcal{N}_{BS}) = \left\{ x \in \mathbb{R}^2 : |x - Y^j| \le |x - Y^k| \text{ for all } k \right\}.$$

Note that with probability one no point of $\mathcal{N}_M$ is shared by two or more BS's.

The Voronoi model of assignment of the active BS to each mobile, although simple, seems to be quite reasonable from the DACP/DCCP point of view. For instance, in a simplified case when $C_i^j = C_i$ (i.e, when the throughput required by the mobile does not depend on the BS that serves it), when $\kappa_i \equiv \kappa$ and when the path loss function l is monotonic in the Euclidean distance, then the choice of the nearest BS minimizes the cost f_i^j (4.2) of acceptance of the user by a BS. We now give two negative results.

Proposition 7.2 *For the Poisson-Voronoi model, (3.7) does not hold for some j with probability 1.*

Proof: By independent marking and the spatial mixing of $\mathcal{N}_M^j$. ■

This means that with probability 1, we will find a BS that has too many mobiles to be able to solve its local power allocation problem.

In order to proceed with our Poisson-Voronoi model we first have to reduce each pattern $\mathcal{N}_M^j$ that violates (3.7).

R1. For each j, let $\overline{\mathcal{N}}_M^j$ be a maximal subset of $\mathcal{N}_M^j$ s.t.

$$\sum_{X_i^j \in \overline{\mathcal{N}}_M^j} H_i^j < 1/\kappa_j \,. \tag{7.3}$$

Of course this maximal set is not uniquely defined (see the discussion on reduction schemes in §4.2). Our only assumption here will be that the reduction policy leads to some stationary ergodic sequence $\{(Y^j, \overline{\mathcal{N}}_M^j, \{C_i^j, W_i^j\}_i, \kappa_j)\}_j$. As shown by the following proposition, this still does not lead to a feasible configuration of mobiles either.

Proposition 7.3 *Assume the Poisson-Voronoi model with maximal patterns of mobiles $\{\overline{\mathcal{N}}_M^j\}$ given by R1. The (deterministic) convergence radius R of matrix $\mathbb{A}$ with entries calculated with respect to $\overline{\mathcal{N}}_M^j$ is strictly less than 1 (in fact for this Poisson-Voronoi model, $R = 0$ a.s.).*

Proof: With probability 1 the Poisson point process $\{(Y^j, \overline{\mathcal{N}}_M^j, \{C_i^j, W_i^j\}_i, \kappa_j)\}_j$ has a cluster of points (BS's) that leads to a super-stochastic block of matrix $\mathbb{A}$. Thus by Proposition 3.4, the convergence radius of $\mathbb{A}$ is larger than 1. In fact, for arbitrary large x we can find a block with line-sums greater than x and this shows that $R = 0$. ■

So under Poisson assumptions, any local ergodic R1 reduction of $\{\mathcal{N}_M^j\}_j$ leads to a pattern $\{\overline{\mathcal{N}}_M^j\}_j$ that is almost surely not feasible.

This surprising property reveals some feature of the Poisson-Voronoi model in an infinite plane: the possible clustering of the BS's allowed by the Poisson model renders the global power allocation problem unfeasible whatever the parameters of the model. In other words, even for very low density of mobiles or very high density of BS, some admission control should be enforced.

7.2.3 The Poisson-Voronoi Model under DACP

In this section we consider the Poisson Voronoi model assuming each BS applies DACP. As previously we assume that the maximal subset $\widetilde{\mathcal{N}}_M^j$ in DACP is obtained by means of some reasonable reduction policy, that leads to a stationary ergodic sequence $\{(Y^j, \widetilde{\mathcal{N}}_M^j, \{C_i^j, W_i^j\}_i, \kappa_j)\}_j$. Then the convergence radius of $\mathbb{A}$ is $R > 1$ and this means $\widetilde{\mathcal{N}}_M^j$ is almost surely feasible. Then DACP naturally leads to the following notion of *admission load*:

(Admission-Load of the Poisson Voronoi Model): For a given $\lambda_{BS} > 0$ and $\epsilon > 0$ let $\lambda_M^\epsilon = \lambda_M^\epsilon(\lambda_{BS})$ be the maximal intensity of $\mathcal{N}_M$ s.t.

$$\Pr\Big(\textit{inequality } (4.3) \textit{ holds for } j = 0\Big) \geq 1 - \epsilon\,. \tag{7.4}$$

The function λ_M^ϵ has the following interpretation. Suppose each BS applies DACP, which leads to a feasible global and local power allocation. For a given BS (say BS 0) and for any intensity of mobiles $\lambda_M > 0$ there is a positive probability that $\widetilde{\mathcal{N}}_M^0 \subsetneq \mathcal{N}_M^0$, meaning that at least one mobile of the initial pattern $\mathcal{N}_M^0$ has to be rejected. Then, λ_M^ϵ is the maximal mobile intensity that makes the probability of outage of at least one mobile of this (or any other) BS less than ϵ.

We postpone the calculation of the probability in (7.4) to § 8. As we shall see there, closed form formulas are known for the expectations of the left-hand-side (lhs) of (4.3), which will be used in the next subsection.

7.2.4 Homogeneous Poisson-Voronoi Mean Model

One of the consequences of the stationarity of $\mathcal{N}_{BS}$ is that the expected values of the random coefficients of the operator $\mathbb{A}$ and vector $\mathbf{b}$ given by (3.9)–(3.10) and (3.11) are equal for each column. Moreover $\{S_j\}$ should form another stationary marking of the point process. The Poisson-Voronoi Mean Model consists in simplifying the problem (3.3) by replacing the random coefficients by their means. We then get the following inequality on (the deterministic) power S: $S \geq \sum_k \mathrm{E}[a_{0k}]S + \mathrm{E}[b_0]$. This has a finite solution if $\sum_k \mathrm{E}[a_{0k}] < 1$, which gives

$$\mathrm{E}\left[\kappa_0 \sum_{i \in \mathcal{N}_M^0} H_i^0 + \gamma \sum_{k \neq 0} \sum_{i \in \mathcal{N}_M^0} \frac{H_i^0 L(Y^k - X_i^0)}{L(0 - X_i^0)}\right] \leq 1. \tag{7.5}$$

This condition is the mean-value version of Condition (4.3) and, as we shall see, it gives load estimates different from (7.4).

8 Exact Formulas Bounds and Approximations for DACP/DCCP

In this section, we concentrate on the Poisson-Voronoi model of § 7.2.2and algebraically analyze its admission load under DACP and DCCP.

Consider a Poisson point process $\mathcal{N}_{BS} = \{Y^j\}$ of the BS's, with intensity λ_{BS} and assume that there is a BS located at $Y_0 = 0$. Let $V_0 = V_0(\mathcal{N}_{BS})$ be the Voronoi cell of point $Y_0 = 0$. Also assume a Poisson process $\mathcal{N}_M = \{X_i\}$ of mobiles, with intensity λ_M.

Our main task is to find the probability of the event (4.3) for $j = 0$. Denote for convenience its complement by

$$\mathcal{E}_0 = \left\{ \kappa_0 \sum_{X_i \in V_0} H_i^0 + \gamma \sum_{k \neq 0} \sum_{X_i \in V_0} \frac{H_i^0 L(Y^k - X_i)}{L(0 - X_i)} > 1 \right\}. \tag{8.1}$$

More specifically we want to approximate the function $\Pr(\mathcal{E}_0) = \Pr(\mathcal{E}_0)(\lambda_M)$ (near the origin) in order to be able to find, for any given $\epsilon > 0$, the maximal λ_M s.t. $\Pr(\mathcal{E}_0) < \epsilon$. Our main tools are explicit formulas for expectations, bounds for the Laplace transforms and some simulation techniques.

8.1 Expectations

First we give closed form formulas of the expectations of the terms in the right-hand-side of (8.1) obtained via Neveu's exchange formula (see [2]). At the same time they allow for calculation of $\mathrm{E}[a_{jj}]$, $\mathrm{E}[a_{jk}]$ and $\mathrm{E}[b_j]$. We will adopt from now on a typical assumption that $L(y,x) = L(|y-x|)$ (with a little abuse of notation); i.e., the path loss only depends on the Euclidean distance. We have

$$\mathrm{E}\left[\kappa_0 \sum_i H_i^0\right] = \frac{\lambda_M}{\lambda_{BS}} \mathrm{E}[\kappa H]\,, \tag{8.2}$$

$$\mathrm{E}\left[\sum_i \frac{H_i^0 W_i}{L(|X_i|)}\right] = \lambda_M 2\pi\, \mathrm{E}[HW] \int_0^\infty \frac{r e^{-\lambda_{BS}\pi r^2}}{L(r)}\, dr \tag{8.3}$$

$$\begin{aligned} \mathrm{E}\left[\sum_{k\neq 0}\sum_i \frac{H_i^0 L(|X_i - Y^k|)}{L(|X_i|)}\right] &= \lambda_M \lambda_{BS} 4\pi^2\, \mathrm{E}[H] \\ &\quad \times \int_0^\infty \left\{ \frac{r e^{-\lambda_{BS}\pi r^2}}{L(r)} \int_r^\infty u L(u)\, du \right\} dr\,, \end{aligned} \tag{8.4}$$

where $\kappa, (C, W, T)$ are generic random variables.

The integrals in the above formulas can be analytically evaluated for some particular attenuation functions. For example, if $L(r) = (A\max(r_0, r))^{-\alpha}$ for some $A > 0$, $r_0 > 0$ and $\alpha > 2$, then

$$\begin{aligned} (8.3) &= \mathrm{E}[HW]\frac{\lambda_M A^\alpha}{\lambda_{BS}} \\ &\quad \times \left(\frac{\Gamma(1+\alpha/2, r_0^2 \lambda_{BS}\pi)}{(\lambda_{BS}\pi)^{\alpha/2}} - r_0^\alpha \left(1 - e^{-r_0^2 \lambda_{BS}\pi}\right)\right) \\ (8.4) &= \mathrm{E}[H]\frac{\lambda_M}{\lambda_{BS}(\alpha-2)} \\ &\quad \times \left(\alpha e^{-r_0^2\lambda_{BS}\pi} + \lambda_{BS}\pi r_0^2 \alpha - \alpha + 2 \right), \end{aligned}$$

where $\Gamma(a,z)$ is the incomplete gamma function $\Gamma(a,z) = \int_z^\infty e^{-t} t^{a-1}\, dt$.

If $L(r) = (1+Ar)^{-\alpha}$ for some $A > 0$ and $\alpha > 2$, then

$$\begin{aligned}(8.4) = \; & \mathrm{E}[H]\frac{\lambda_M}{\lambda_{BS}^{3/2}A^2(\alpha-1)(\alpha-2)} \\ & \times\left(\alpha A\lambda_{BS}\pi + 2\alpha A^2\lambda_{BS}^{1/2} - 2A^2\lambda_{BS}^{1/2} + 2\pi\lambda_{BS}^{3/2}\right)\end{aligned}$$

and for $\alpha = 3$,

$$(8.3) = \frac{\mathrm{E}[HW]\lambda_M(3A^3 + 6A\pi\lambda_{BS} + 4\pi\lambda_{BS}^{3/2} + 12A^2\lambda_{BS}^{1/2})}{4\lambda_{BS}^{5/2}\pi}.$$

A formula, albeit complicated, can be obtained for general α too. Notice that for both forms of the attenuation function L, for large A and small r_0 (eg. $A \approx 1000$, $r_0 \approx 1/1000$) and for reasonable values of λ_{BS} (i.e. $\lambda_{BS} < 10\,BS/\mathrm{km}^2$, $\lambda_M < 100$, mobiles $/\mathrm{km}^2$), the above expectations can be well approximated by the following formulas

$$\begin{aligned}(8.3) \quad &\approx\quad \mathrm{E}[HW]\frac{\lambda_M A^\alpha}{\lambda_{BS}}\frac{\Gamma(1+\alpha/2)}{(\lambda_{BS}\pi)^{\alpha/2}} \\ (8.4) \quad &\approx\quad \mathrm{E}[H]\frac{2\lambda_M}{\lambda_{BS}(\alpha-2)},\end{aligned}$$

where $\Gamma(a)$ is the complete gamma function $\Gamma(a) = \int_0^\infty e^{-t}t^{\alpha-1}\,dt$, which correspond to a simplified attenuation function $L(r) = (Ar)^{-\alpha}$. It follows that the ratio of the other-cell-interference (8.4) to the other-cell-interference (8.2) is approximately $2/(\alpha-2)$. This value coincides with the analogous ratio calculated in [4] for the M/M CDMA uplink model.

Finally, we remark, that the equation $(8.2) + (8.4) = 1$ can easily be solved in λ_M, and this gives ann explicit formula for the (DACP) load of the M/M mean model of §7.2.4.

8.2 Bounds

The expectations calculated in the previous section give bounds on other capacities. The most direct one (and rather crude) is based on the Markov inequality: $\Pr(\mathcal{E}_0) \le (8.2) + (8.4)$. Thus the solution in λ_M of the equation $(8.2) + (8.4) = \epsilon$ is a very conservative (but explicit) bound of the Admission-Load of § 7.2.3.

Suppose $\kappa_0 = const$; by Jensen's inequality, the mean SIR $\mathcal{C}^0$ in (5.5) can be bounded by

$$\mathrm{E}[\mathcal{C}^0] \ge 1/\big(\kappa_0(8.2)' + (8.4)' - \kappa_0\Pr(\#\mathcal{N}_M^0 > 0)\big),$$

where $(8.2)'$ and $(8.4)'$ are given by formulas, (8.2) and (8.4) respectiveley, with the factor $\mathrm{E}[H]$ suppressed. This bound can be made explicit because $\Pr(\#\mathcal{N}_M^0 > 0) \ge \lambda_M/(\lambda_M + \lambda_{BS})$.

If $\mathcal{C}^0$ is small enough for justifying the approximation $\log(1+\mathcal{C}^0) \sim \mathcal{C}^0$, then one deduces from the last bounds and from (5.6) that under DCCP,

$$\mathrm{E}[\mathcal{B}^0] \ge B/\big(\kappa_0(8.2)' + (8.4)' - \kappa_0\Pr(\#\mathcal{N}_M^0 > 0)\big).$$

More precise bounds of the distribution function of the lhs of (4.3), can be obtained via Chernov's inequality $\Pr(\text{lhs of } (4.3) \ge z) \le \inf_{\theta>0}\ \mathrm{E}[\exp\{-\theta(z - [\text{lhs of } (4.3)])\}]$. Although the

Laplace transform of the lhs of (4.3) is not known in explicit form, it can be shown by Jensen's inequality that

$$\begin{aligned}
&\mathrm{E}\Big[e^{\theta[\text{lhs of (4.3) for } j=0]}\Big|\#\mathcal{N}_M^0 = N\Big] \\
&\leq \mathrm{E}\left[\exp\left\{\theta N H_0\left(\kappa_0 + \gamma \sum_{k\neq 0} \frac{L(|Y^k - X_0^*|)}{L(|X_0^*|)}\right\}\right)\right],
\end{aligned}$$

where X_0^* is a point uniformly chosen within the cell $V(0)$. This inequality corresponds to the situation when N users originally independently distributed in $V(0)$ are gathered in one "hot spot" at X_0^*. If $L(r) = (Ar)^{-\alpha}$, then the right hand side of the last inequality can be bounded in an explicit way that results in the following LDP-type bound

$$\begin{aligned}
&\Pr\Big([\text{lhs of (4.3) for } j=0] \geq z \Big| \#\mathcal{N}_M^0 = N\Big) \\
&\leq \inf_{\theta>0} \mathrm{E}\left[\frac{\lambda_{BS}\,\mathrm{E}^{1/2}[|V(0)|^2]e^{-\theta(z-N\kappa)}}{1-J(2\theta N\gamma)}\right],
\end{aligned} \tag{8.5}$$

where $J(t) = t^{2/\alpha}\int_{t^{-1/\alpha}} t(e^{t^{-\alpha}}-1)\,dt$, and $\inf_{\theta>0}$ is taken over $\theta:\ 0 < J(...) < 1$ (see [2] for the details). Note that (8.5) can be used for the bounding of $\Pr(\mathcal{E}_0)$, or for that of $\Pr(\mathcal{C}^0 \leq c)$ for small c, or equivalently for that of $\Pr(\mathcal{B}^0 \leq b)$ for small b.

9 Simulation

9.1 Static DACP Load Simulation

We now describe briefly how to get an estimator of $\Pr(\mathcal{E}_0)(\lambda)$ from simulation. For simplicity we concentrate on the homogeneous case. We choose a discrete set of test intensities (of mobiles) $\lambda_0 < \lambda_1 < \ldots \lambda_k$ and simulate k independent patterns of Poisson point processes $\mathcal{N}_i$ $(i = 0, \ldots, k)$ with respective intensities λ_0 and $\Delta_i = \lambda_i - \lambda_{i-1}$ in the Voronoi cell $V(0)$ generated by a given pattern $\mathcal{N}_{BS}$. Let F^i be the event that (8.1) holds for $\mathcal{N}_M = \sum_{j=0}^{i}\mathcal{N}_M^j$. Obviously $\Pr[F^i] = \Pr(\mathcal{E}_0)(\lambda_i)$ and F^i is increasing in i. The same holds for $F^i_{(n)} = 1/n\sum_{u=1}^{n} F^{i,u}$, where $(F^{i,u}, i = 0, \ldots, k)$, $u = 1, \ldots, n$ are independent copies of $(F^i, i = 0, \ldots, k)$. In addition, $F^i_{(n)}$ converges a.s. to $\Pr[F^i]$ as $n \to \infty$.

9.2 Dynamic DACP Load Simulation

The static DACP load simulation, considered previously yields estimates on the probability of no-outage (or the complement: at least one outage) in a typical cell. This is a BS-centered QoS parameter. Now we show how to draw conclusions concerning the pattern of users accepted in a long run of DACP.

For each given realization of the process of BS's $\mathcal{N}_{BS}$, we want to simulate a spatio-temporal process $\widetilde{\mathcal{N}}_M|_{V(0)}(t)$ of users in the cell $V(0) = V(0)(\mathcal{N}_{BS} \cup \{0\})$ under Condition (4.3). The evolution of $\widetilde{\mathcal{N}}_M|_{V(0)}(t)$ is such as in a conditional birth-and-death process: points are generated at exponential periods (with an exponential distribution parameter equal to $\Lambda = \int_{V(0)} \lambda_M(dx)$) and located in $V(0)$ with the distribution $\lambda_M(\cdot)/\Lambda$, but only if their presence does not violate (4.3). Points located in $V(0)$ stay there for exponential times (with parameter 1) and are then removed. A long run of $\widetilde{\mathcal{N}}_M|_{V(0)}$ yields a (temporal) steady state of those points of the Poisson point process with intensity measure $\lambda_M(\cdot)$, which are accepted under DACP. A more sophisticated scheme of exact simulation of the steady state of $\widetilde{\mathcal{N}}_M|_{V(0)}$, similar to this proposed in [12], can be employed too (see [18]).

We repeat the above simulation of the (almost or exact) steady state of $\widetilde{\mathcal{N}}_M|_{V(0)}$ for many realizations of $\mathcal{N}_{BS}$ and get the estimates on the density of the process of users accepted under DACP.

9.3 DCCP Load Simulation

DCCP load estimations are based on the simulation of $\mathcal{N}_{BS}$ and $\mathcal{N}_M$ in $V(0)(\mathcal{N}_{BS}\cup\{0\})$. From this, we obtain samples of the SINR ratio C^0 given by (5.5).

10 Discussion of the Numerical Results

The default assumptions of the model that we study are as follows:

- attenuation function: $L(r) = (1 + 1000r)^{-3}$, r in km,
- $\lambda_{BS} = 0.1768\, BS/\mathrm{km}^2$,
- $C = 0.011797$, $\kappa = 0.2$, $\gamma = 1$.

We begin our study by the mean model. Inequality (7.5), together with the explicit form of the expectations given in § 8.1, yield an explicit bound on the average number λ_M of mobiles per km^2 that can be served by a network of λ_{BS} BS's per km^2 (see Figure 1). Note that the dependance in $\lambda_{BS} < 10$ is nearly linear, with a slope of appr. 38.54 Mobiles/BS, for all three attenuation functions considered in § 8.1. For larger values of λ_{BS}, the shape of the attenuation close to the antenna becomes important for the evaluation of load. The curve for $L(r) = (Ar)^{-\alpha}$ remains linear, while the two attenuations that are bounded at the origin give ultimately bounded capacities: there is an upper bound (which is larger for the attenuation $L(r) = (1 + Ar)^{-\alpha}$) on the number of mobiles that can be served on 1 km^2, whatever the density of BS's. This effect can of course be neglected in present CDMA networks, but it should probably be taken into account for planning more dense networks.

In order to estimate the stochastic load under DACP, we used the simulation schemes described in § 9. Figure 2 [a] shows the estimates of the outage probability obtained via the static DACP scheme of § 9.1 for the M/M model (the flater curve) and the H/M model (steeper curve). The straight line $0.1465\lambda_M$ corresponds to the expectation in (7.5), i.e. to the mean model, wich gives a load 6.824 mobiles per km^2 or 38.54 mobiles per BS. Figure 2 [b] compares the simulated value of the static DACP outage probability (per cell) for M/M model to the Markov (linear) and Chernov (exponential) bound via (8.5) and the simulated value.

Now we consider the dynamic scheme of § 9.2. Figure 3 [a] shows the density of users $\widetilde{\lambda}_M(r)$ accepted in the long run by DACP as a function of the distance r to the BS (lower curve). For comparison, the upper line represents the density $\lambda^V_M(r)$ of all users who apply for this BS (all points of the Poisson process in the cell) and the flat line is the constant density of all users on the plane corresponding to $\lambda_M = 0.5$ users /km^2. Note that the ratio $\widetilde{\lambda}_M(r)/\lambda^V_M(r)$ is an estimator of the probability of the acceptance of a customer within the distance r to the BS

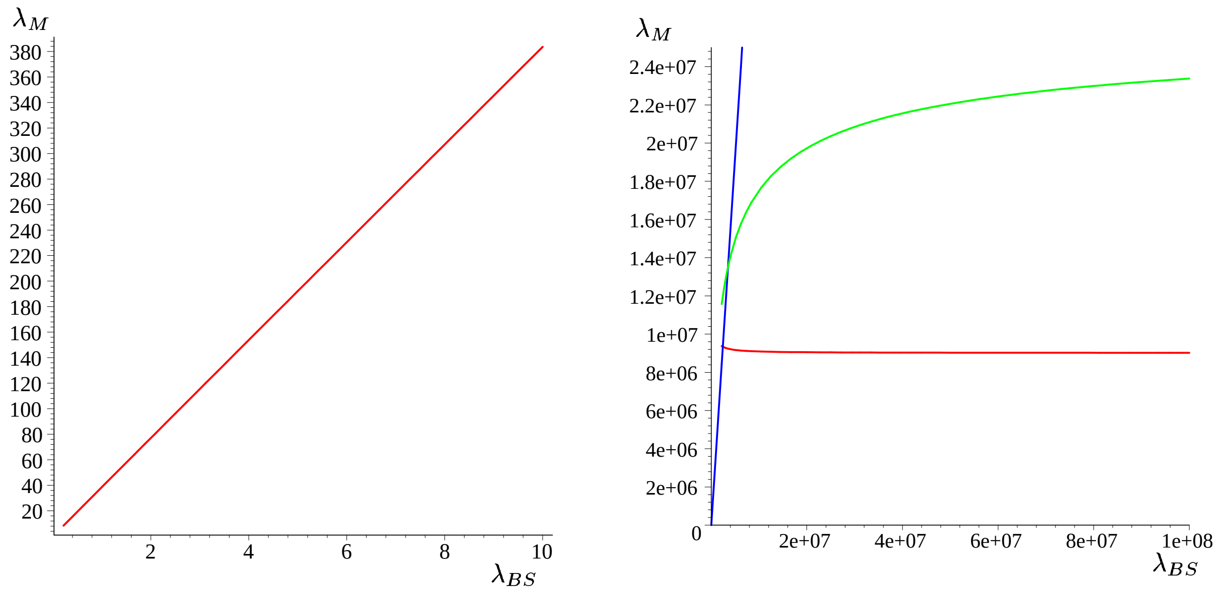


Figure 1: Maximal intensity λ_M of mobiles per km^2 and $\rho = \lambda_M/\lambda_{BS}$ of mobiles per base-station satisfying (7.5) for M/M model; the real and extremely dense case.

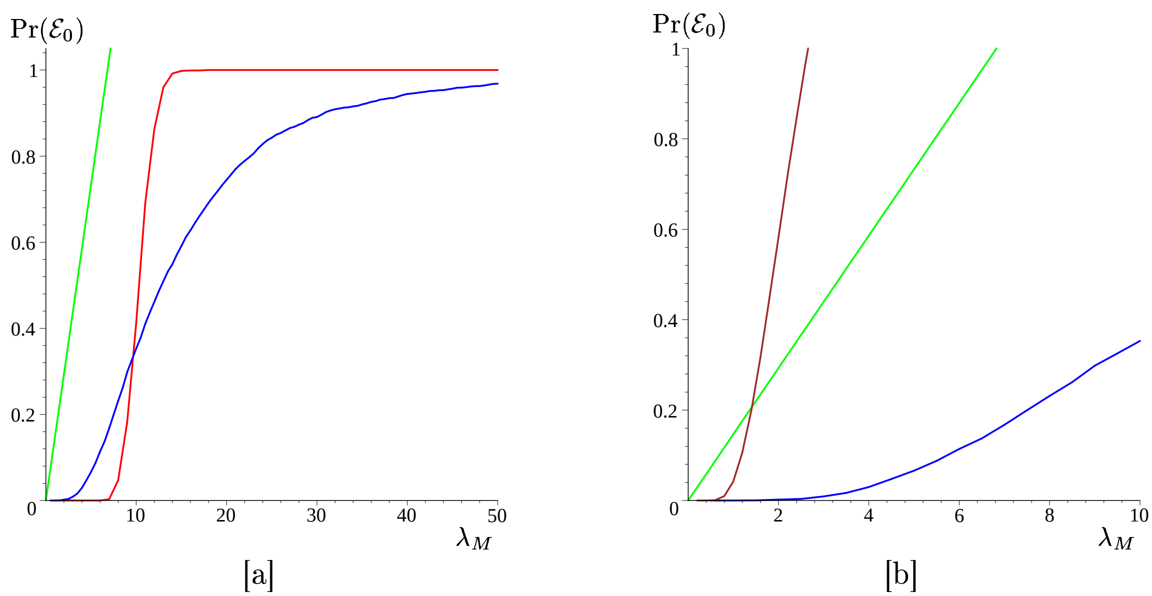
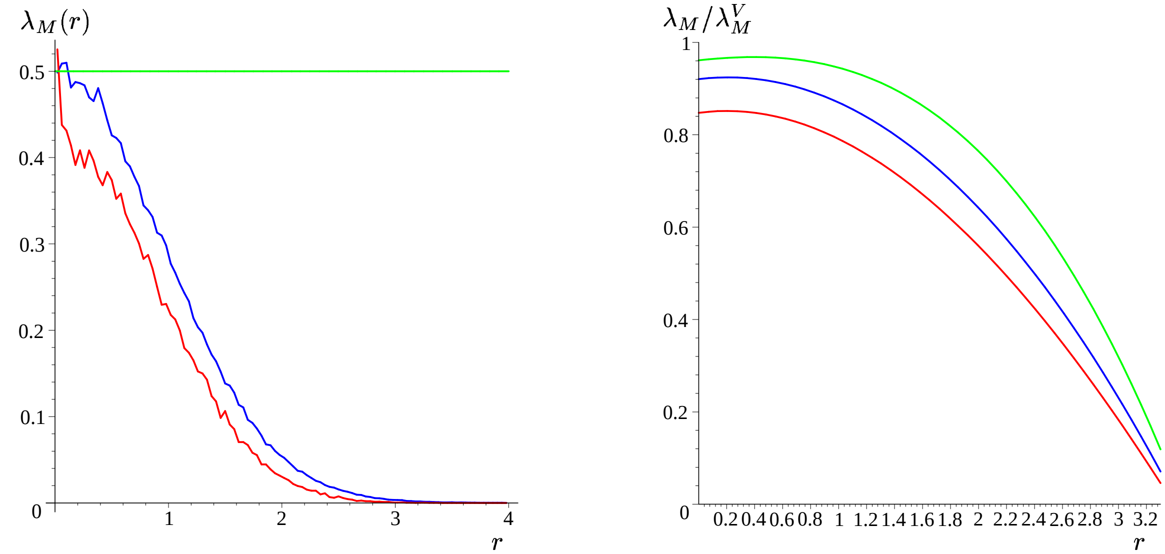


Figure 2: [a] Outage probability (per cell) for M/M model (more flat curve) and H/M model (more steep) curve. The straight line corresponds to the expectation in (7.5). [b] Markov bound, Chernov bound, and the simulated value for M/M model.

by DACP in the stationary regime. Figure 3 [b] shows these estimates for initial intensities $\lambda_M = 0.5$, $\lambda_M = 0.04$ and $\lambda_M = 0.024$ (from bottom to top). Figure 4 shows the mean load $\mathrm{E}[C^0]$ under DCCP as a function of the intensity of users λ_M estimated by the simulation scheme of (see § 9.3) and its explicit lower bound (see § 8.2).



[a] [b]

Figure 3: [a] Density of users $\widetilde{\lambda}_M(r)$ accepted in the long run by DACP as a function of distance to BS (lower curve). [b] Estimates of the probability of the rejection of a customer within the distance r to the BS by DACP.

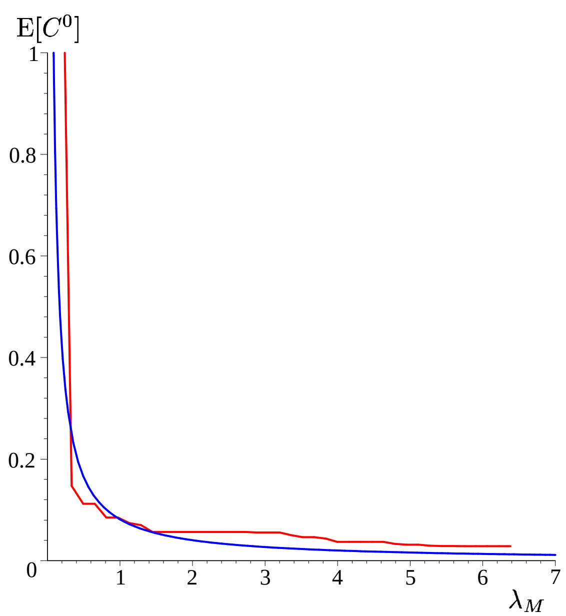


Figure 4: The mean load $\mathrm{E}[C^0]$ under DCCP as a function of the intensity of users λ_M.

Conclusions, Future Work

This paper shows that the algebraic approach to power control leads to scalable admission and congestion control algorithms for large CDMA networks. In contrast to most studies, the geometry of the inter-cell interference is represented in an exact way, and not via a fraction of the own-cell interference. Stochastic geometry was used to prove that these algorithms yield a positive load in infinite networks and to give estimates of the mean values and the fluctuations of load within this context. In particular, the analysis using the mean Poisson spatial model offers explicit formulas for the maximal traffic that can be served. Moreover, this approach allows one to address the interplay between several traffic classes (with fixed and elastic bit rates). (For other papers using stochastic geometry for networks, see eg. [1, 3, 7].)

Future work will concentrate on the extension of this approach to the joint analysis of the downlink and the uplink with maximal power constraints.

Appendix. Proofs of the statements of section 3

A.1 Block structure of $\tilde{\mathbb{A}}$

Note that (3.1) is equivalent to the following multidimensional linear inequality on the variables $\{S_i^j\}_{i,j}$:

$$\begin{aligned}&\text{for all } i,j\\ &S_i^j \ge \frac{C_i^j}{l(Y^j,X_i^j)}\Big(W_i^j + \kappa_j l(Y^j,X_i^j)(P_j + \sum_{i'\neq i} S_{i'}^j)\\ &\qquad\qquad + \gamma \sum_{k\neq j} l(Y^k,X_i^j)(P_k + \sum_{i'} S_{i'}^k)\Big)\,.\end{aligned} \tag{A.1}$$

Denote by $N^j = \#\mathcal{N}_M^j$ the number of mobiles served by Y^j. For each i,j, we add to both sides of (A.1) $\kappa_j C_i^j S_i^j$, divide by $1+\kappa_j C_i^j$ and we obtain the set of inequalities equivalent to the matrix inequality (3.2) where $\tilde{\mathbf{S}}$ is the unknown vector of dimension $1 \times \sum_{k=1}^{\#\mathcal{N}_{BS}} N^k$ (i.e., it has one coordinate per user), $\tilde{\mathbb{A}}$ is $\sum_{k=1}^{\#\mathcal{N}_{BS}} N^k \times \sum_{k=1}^{\#\mathcal{N}_{BS}} N^k$ matrix and $\tilde{\mathbf{b}}$ is $1 \times \sum_{k=1}^{\#\mathcal{N}_{BS}} N^k$. The matrix $\tilde{\mathbb{A}}$ and the vectors $\tilde{\mathbf{b}}$ have the following block structure

$$\begin{aligned}\tilde{\mathbb{A}} &= \Big(\tilde{\mathbb{A}}_{jk}\Big)_{jk=1}^{\#\mathcal{N}_{BS}}\\ \tilde{\mathbf{b}} &= \Big(\tilde{\mathbf{b}}_k\Big)_{k=1}^{\#\mathcal{N}_{BS}}\\ \tilde{\mathbf{S}} &= \Big(\tilde{\mathbf{S}}_k\Big)_{k=1}^{\#\mathcal{N}_{BS}},\end{aligned}$$

i.e., one block per BS (per pair of BS's for the matrix). The block $\tilde{A}_{jk}$ has dimension $N^j \times N^k$ and is constant in lines

$$\tilde{\mathbb{A}}_{jk} = \boldsymbol{\tau}_{jk}\mathbf{1}_k^t$$

where $\boldsymbol{\tau}_{jk}$ is $1 \times N^j$ vector

$$(\boldsymbol{\tau}_{jk})_i = \begin{cases} \dfrac{\gamma C_i^j l(Y^k,X_i^j)}{(1+\kappa_j C_i^j)(Y^j,X_i^j)} & \text{for } k \neq j\\[2ex] \dfrac{\kappa_j C_i^j}{1+\kappa_j C_i^j} & \text{for } k = j\end{cases}$$

$$(i = 1,\dots,N^j)$$

and $\mathbf{1}_k$ is $1 \times N^k$ vector with all coordinates 1, and $()^t$ means the transposition. Similarly, $\tilde{\mathbf{b}}_j$ is the vector of dimension $1 \times N^j$ with entries

$$\begin{aligned}(\tilde{\mathbf{b}}_j)_i &= \frac{C_i^j}{l(Y^j, X_i^j)(1+\kappa_j C_i^j)} \\ &\quad \times \Big(W_i^j + \kappa_j l(Y^j, X_i^j) P_j + \gamma \sum_{k \neq j} l(Y^k, X_i^j) P_k \Big) \\ &\qquad (i = 1, \dots, N^j)\end{aligned}$$

and $\tilde{\mathbf{S}}_j = \big(S_i^j\big)_{i=1}^{N^j}$.

A.2 Proof of Proposition 3.1

Denote by $S_j = \sum_{i=1}^{N^j} S_i^j$ the total power emitted by BS j on the traffic channels toward its users. Note that in our notation $S_j = \mathbf{1}_j^t \tilde{\mathbf{S}}_j$. The inequality (3.2) for the jth block of $\tilde{\mathbf{S}}$ reads

$$\begin{aligned}\tilde{\mathbf{S}}_j &\geq \sum_k \tilde{\mathbb{A}}_{jk} \tilde{\mathbf{S}}_k + \tilde{\mathbf{b}}_j \\ &= \sum_k \boldsymbol{\tau}_{jk} \mathbf{1}_k^t \tilde{\mathbf{S}}_k + \tilde{\mathbf{b}}_j \\ &= \sum_k \boldsymbol{\tau}_{jk} S_k + \tilde{\mathbf{b}}_j\,.\end{aligned} \tag{A.2}$$

Multiplying both sides by $\mathbf{1}_j^t$ from the left (equivalent to adding coordinates of the vectors) we get

$$\begin{aligned}S_j &\geq \sum_k \mathbf{1}_j^t \boldsymbol{\tau}_{jk} S_k + \mathbf{1}_j^t \tilde{\mathbf{b}}_j \\ &= \sum_k a_{jk} S_k + b_j\,,\end{aligned} \tag{A.3}$$

where $a_{jk} = \mathbf{1}^t \boldsymbol{\tau}_{jk}$, $b_j = \mathbf{1}^t \tilde{\mathbf{b}}_j$, which gives (3.3) with with $\mathbb{A} = (a_{jk})$, $\mathbf{b} = (b_j)$. Note that a_{jk} and b_j have explicit forms (3.9)–(3.10) and (3.11), respectively.

Coming back to (A.2) we can rewrite it now

$$\tilde{\mathbf{S}}_j \geq \mathbb{A}_{jj} \tilde{\mathbf{S}}_j + \sum_{k \neq j} \boldsymbol{\tau}_{jk} S_k + \tilde{\mathbf{b}}_j\,, \tag{A.4}$$

which is (3.4) with $\tilde{\mathbf{d}}_j = \sum_{k \neq j} \boldsymbol{\tau}_{jk} S_k + \tilde{\mathbf{b}}_j$. Explicit form of (A.4) is given by (3.6).

If we introduce the vector $\mathbf{S} = \big(S_k\big)_{k=1}^{\#\mathcal{N}_{BS}}$ of total powers, the vector $\mathbf{b} = \big(b_k\big)_{k=1}^{\#\mathcal{N}_{BS}}$ and the matrix $\mathbb{A} = \big(a_{jk}\big)_{kj=1}^{\#\mathcal{N}_{BS}}$.

In view of (A.3) and (A.4), the matrix inequality (3.2) is equivalent to the following two-step problem: solving the inequality

$$\mathbf{S} \geq \mathbb{A}\mathbf{S} + \mathbf{b} \tag{A.5}$$

for the total powers emitted by the BS's and for given solution $\mathbf{S} = \big(S_j\big)_j$ of (3.3), for each j to solve the inequality for the jth block

$$\tilde{\mathbf{S}}_j \geq \tilde{\mathbb{A}}_{jj}\tilde{\mathbf{S}}_j + \tilde{\mathbf{d}}_j\,, \tag{A.6}$$

where $\tilde{\mathbf{d}}_j = \sum_{k\neq j}\boldsymbol{\tau}_{jk}S_k + \tilde{\mathbf{b}}_j$ under the constraint

$$\mathbf{1}_j^t\tilde{\mathbf{S}}_j = S_j\,. \tag{A.7}$$

A.3 Proof of Proposition 3.2

Multiplying (3.4) from the left by $\mathbf{1}_j^{(t)}$, as in (A.3), we get $S_j \geq \mathbf{1}_j^{(t)}\boldsymbol{\tau}_{jj}S_j + \mathbf{1}_j^{(t)}\tilde{\mathbf{d}}_j$, thus necessarily $\mathbf{1}_j^{(t)}\boldsymbol{\tau}_j \equiv a_{jj} < 1$, which is equivalent to (3.7). Now, inequality (3.4) is equivalent to $(\mathbb{I}-\tilde{\mathbb{A}}_{jj})\tilde{\mathbf{S}}_j \geq \tilde{\mathbf{d}}_j$, where $\mathbb{I}$ is the respective identity matrix. The matrix $(\mathbb{I}-\tilde{\mathbb{A}}_{jj})$, under constraint $a_{jj} < 1$ has its inverse $(\mathbb{I}-\tilde{\mathbb{A}}_{jj})^{-1} = \mathbb{I} + \boldsymbol{\tau}_{jj}\mathbf{1}_j^{(t)})/(1-a_{jj})$, thus any solution of (3.4) has the form $\tilde{\mathbf{S}}_j = \mathbb{I} + \boldsymbol{\tau}_{jj}\mathbf{1}_j^{(t)}(\tilde{\mathbf{d}}_j + \boldsymbol{\xi})/(1-a_{jj})$, where $0 \leq \boldsymbol{\xi} = (\xi_i)_{i=1}^{\#\mathcal{N}_M^j}$. The constraint $\sum_i S_i^j \equiv \mathbf{1}_j^{(t)}\tilde{\mathbf{S}}_j = S_j$ makes

$$\begin{aligned}
\sum_i \xi_i &\equiv \mathbf{1}_j^{(t)}\boldsymbol{\xi} \\
&= S_j(1-a_{jj}) - \mathbf{1}_j^{(t)}\tilde{\mathbf{d}}_j \\
&= S_j(1-a_{jj}) - \sum_{k\neq j} S_k a_{jk} - b_j \\
&= S_j 1 - \sum_k S_k a_{jk} - b_j\,.
\end{aligned}$$

Note in the above that $\mathbf{1}^{(t)}(\tilde{\mathbf{d}}_j + \boldsymbol{\xi})/(1-a_{jj}) = S_j$. Substituting this to our solution we get $\tilde{\mathbf{S}}_j = \boldsymbol{\tau}_{jj}S_j + \tilde{\mathbf{d}}_j + \boldsymbol{\xi}$ that is equivalent in explicit form to (3.8). This completes the proof.

A.4 Proof of Proposition 3.3

We recall first the standard theory on the matter. We say that $\mathbb{A}$ is *irreducible* if for each pair of j,k exist $n \geq 1$ such that $a_{jk}^n > 0$. If $\mathbb{A}$ is irreducible then the power series $A_{jk}(z) = \sum_{n=0}^{n} a_{jk}^n z^n$ for $j,k = 1,2,\ldots$ all have a common convergence radius $0 \leq R < \infty$, for each pair k,j. We will call R the *convergence radius* of $\mathbb{A}$. The reciprocal $1/R$ is called the Perron value of $\mathbb{A}$; if $\mathbb{A}$ is finite it is its *Perron-Frobenius eigenvalue* (or *spectral radius*). Moreover $A_{jk}(R) < \infty$ for all $j \neq k$, and $A_{jj}(R)$ is finite or infinite at the same time for all j making $\mathbb{A}$ respectively *transient* or *recurrent*. Define the matrices $\mathbf{F}^n = (f_{jk}^n)_{jk}$ recursively for $n \geq 0$ by putting $\mathbf{F}^0 = 0$, $\mathbf{F}^1 = \mathbb{A}$ and $f_{jk}^{n+1} = \sum_{r\neq k} a_{jr} f_{rk}$ for $n \geq 1$. Finally let $\mathbf{F}^* = (f_{jk}^*)_{jk} = \sum_{n=0}^{\infty}\mathbf{F}^n$. Now, Proposition 3.3 follows from the following classical results on Perron-Frobenius theory (for details the reader may consult [17] chapter 6 or [13] chapter 7.) Suppose $\mathbb{A}$ is nonnegative irreducible matrix and let $\mathbf{b}$ be nonnegative vector.

i. If $R > 1$ then $\mathbb{A}^*\,\mathbf{b}$ is the minimal nonnegative solution of the problem $\mathbf{s} = \mathbb{A}\mathbf{s} + \mathbf{b}$; any other nonnegative solution is of the form $\mathbb{A}^*\,\mathbf{b} + \mathbf{z}$, with nonnegative $\mathbf{z}$ such that $\mathbb{A}\mathbf{z} = \mathbf{z}$. Moreover if matrix $\mathbb{A}$ is finite then it is the

ii. If $R = 1$ and $\mathbb{A}$ is transient then the thesis of (i) holds true.

iii. If $R = 1$ and $\mathbb{A}$ is recurrent then each nonnegative solution $\mathbf{s}$ of $\mathbf{s} \geq \mathbb{A}\mathbf{s}$ yields necessarily $\mathbf{s} = \mathbb{A}\mathbf{s}$ and is a scalar multiple of any column of $\mathbf{F}^*$.

iv. If $R < 1$ then there is no nonnegative, finite solution of the inequality $\mathbf{s} \geq \mathbb{A}\mathbf{s}$.

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